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LAMINAR FLOW OF MAGNETORHEOLOGICAL FLUID FLOWING IN CURRENT PIPES

BY

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Abstract. Magnetorheological fluids are Bingham type non-Newtonian fluids. In motion, the kinematic and energetic characteristics of the Bingham type fluid flow differ from the ones of the Newtonian fluid. MR fluids are energized by an external magnetic field influencing the slip stress. Bingham-type MR fluids form a central plug region moving at constant velocity. This paperwork analyzes the effect of external magnetic field on the flow velocity of magnetorheological fluids. The numerical modeling of the phenomenon shows that the magnetic field value has a significant influence on the fluid flow velocity.

Keywords: magneto rheological fluid; plug, velocity; magnetic induction.

1. Introduction

Magnetorheological fluids are Bingham type non-Newtonian fluids that can be classified as smart fluids.

In motion, the kinematic and energetic characteristics of the Bingham type fluid flow differ from the ones of the Newtonian fluid. The variation of the

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unit tangent friction factor is defined by the equation $\tau = \tau_0 + \eta \frac{dV}{dz}$, in which τ_0 is deformation and $\eta \frac{dV}{dz}$ is shear stress.

Magnetorheological fluids are energized by an external magnetic field influencing the slip stress.

Bingham-type magnetorheological fluids form a central plug region moving at constant velocity.

Considering the complex dynamics of the magnetically controlled fluids, it is hard to analyze them fully. To study the flow, mathematical models from the dynamics of real fluids plus mathematical models specific to electromagnetism is employed.

The paperwork shows theoretical results obtained from studying the effect of the intensity of applied magnetic field H, respectively of magnetic induction B on the pressure differences of MR fluid flow in a pipe.

2. Mathematical Models Used in Theoretical Research of Magnetorheological Fluid Flow

MR fluids are non-Newtonian plastic fluids Bingham (Craig, 2003; Siginer *et al.*, 1999; Chilton and Stainsby, 1998) characterized by the relation:

$$\tau = \tau_0 + \eta \frac{dV}{dz} \quad \tau \rangle \tau_0 \tag{1}$$

In which: τ – shear stress, $\tau_0 = \tau_0(B)$ – deformation stress, $\eta = ct.$ – fluid dynamic viscosity.

Considering that the magnetic induction $B = \mu \cdot H$, in which μ is environment permeability [$\mu = \mu_0 \left(1 + \chi\right)$], it results that the total friction of the MR fluid depends on the size of the magnetic induction *B* by the term $\tau_0 \left(B\right)$.

If we assume that by using a coil a magnetic field is obtained, the magnetic induction value depends on the current I induced in N loops of the coil with length *l*, namely $B = B\left(I\right)$.

$$B = \mu \frac{I \cdot N}{l} \tag{2}$$

It results that MR fluid can be controlled if it flows in a circular pipe equipped with a coil run by current *I*. By regulating the intensity of current *I*, the deformation stress values vary.

The Herschel-Bulkley model (Herschel and Bulkley, 1926)

] is employed to calculate the stress τ for magnetic induction values $B > 0$.

$$\tau = \eta\dot{\gamma}\left[1 - \tanh\left(\frac{\dot{\gamma}}{\dot{\gamma}^*}\right)\right] + \left[\tau_0 + \tau_1\left(\frac{\dot{\gamma}}{\dot{\gamma}^*}\right)^{1-n}\right]\tanh\left(\frac{\dot{\gamma}}{\dot{\gamma}^*}\right) \tag{3}$$

For the laminar flow, Chilton and Stainsby (1998) propose a formula (4) to calculate pressure drop. The equation needs an iterative solution

$$\frac{\Delta P}{L} = \frac{4K}{D}\left(\frac{8V}{D}\right)^n\left(\frac{3n+1}{4n}\right)^n\frac{1}{1-X}\left(\frac{1}{1-aX-bX^2-cX^3}\right)^n \tag{4}$$

$$X = \frac{4L\tau_y}{D\Delta P},\ a = \frac{1}{2n+1},\ b = \frac{2n}{(n+1)(2n+1)},\ c = \frac{2n^2}{(n+1)(2n+1)}$$

For the turbulent flow, the authors propose a method that needs knowing wall shear stress, but they do not provide a formula for it. The model was perfected by Hathoot:

$$R = \frac{4n\rho VD\left(1-aX-bX^2-cX^3\right)}{\mu_{perete}(3n+1)} \tag{5}$$

$$\mu_{perete} = \tau_{perete}^{1-1/n}\left(\frac{K}{1-X}\right)^{1/n},\ \tau_{perete} = \frac{D\Delta P}{4L}$$

3. Theoretical Model of Bingham-Type MR Fluid Flow Dynamics in a Circular Pipe under the Influence of an External Magnetic Field

In the case of Bingham type bodies, the kinematic and energy characteristics of the flow differ from those of the Newtonian fluids. According to the variation of the unit tangent friction factor, $\tau = \tau_0 + \eta\frac{dV}{dz}$, the distribution of velocities in the cross section of the pipe includes two sub-fields (Kciuk and Turczyn, 2006). In the central area of r_0 radius, the unit factor has a lower value that the flow limits τ. The fluid travels as a rigid system, apparently non-deformable, like a cylindrical plug. The solid plug travels with a constant velocity with no modification in its geometry.

The value of the r_0 radius, which is the barrier between the two sub-fields, depends on the rheological characteristics of the fluid.

In the sub-field $r \in [r_0; R]$, the stress τ exceeds the value of τ_0 and the character of the flow changes, Fig. 1.

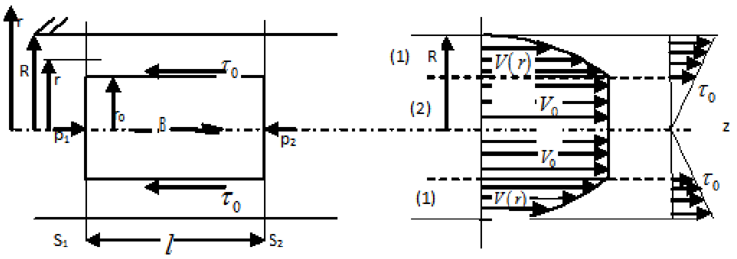

Fig. 1 – Domains of the flow.

The description of the laminar motion of fluids is made by using the Navier-Stokes equations written in cylindrical coordinates (r, φ, z).

$$\begin{gathered}
v_r \frac{\partial v_r}{\partial r} + \frac{v_\varphi}{r}\frac{\partial v_r}{\partial \varphi} + v_z \frac{\partial v_r}{\partial z} - \frac{v^2 \varphi}{r} = f_r - \frac{1}{\varphi}\frac{\partial p}{\partial r} + \vartheta\left(\Delta v_r - \frac{v_r}{r^2} - \frac{2}{r^2}\frac{\partial v_\varphi}{\partial \varphi}\right), \\
v_r \frac{\partial v_\varphi}{\partial r} + \frac{v_\varphi}{r}\frac{\partial v_\varphi}{\partial \varphi} + v_z \frac{\partial v_\varphi}{\partial z} + \frac{v_r v_\varphi}{r} = f_\varphi - \frac{1}{\varphi}\frac{1}{r}\frac{\partial p}{\partial \varphi} + \vartheta\left(\Delta v_\varphi + \frac{2}{r^2}\frac{\partial v_r}{\partial \varphi} - \frac{v_\varphi}{r^2}\right), \\
v_r \frac{\partial v_z}{\partial r} + \frac{v_\varphi}{r}\frac{\partial v_z}{\partial \varphi} + v_z \frac{\partial v_z}{\partial z} = f_z - \frac{1}{\varphi}\frac{\partial p}{\partial z} + \vartheta \Delta v_z
\end{gathered} \tag{6}$$

$$\vec{V}(r,\varphi,z) = v_r(r,\varphi,z)\vec{e_r} + v_\varphi(r,\varphi,z)\vec{e_\varphi} + v_z(r,\varphi,z)\vec{k}$$

$$\Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}. \tag{7}$$

in which $\vec{V}(r,\varphi,z)$ is the flow velocity of the real fluid being in permanent motion with the kinematic viscosity ϑ.

The continuity equation in cylindrical coordinates is:

$$\frac{1}{r}\frac{\partial}{\partial r}(r \cdot v_r) + \frac{1}{r}\frac{\partial v_\varphi}{\partial_\varphi} + \frac{\partial v_z}{\partial x} = 0 \tag{8}$$

It is considered the unidirectional and axially symmetrical flow with the velocity of $\vec{V}$, in a cylindrical pipe with radius R and length l under the action of a pressure gradient. In these conditions, $v_r = 0; v_\varphi = 0; v_z = V \neq 0$.

If mass forces are neglected, $f_r = f_\varphi = f_z = 0$, and continuity equation is used in the required conditions, the equation of dynamic equilibrium results:

$$\eta \frac{d^2V}{dr^2} + \frac{1}{r}\frac{dV}{dr} = \frac{\partial p}{\partial z} \tag{9}$$

Taking into account the expression of the friction factor for non-Newtonian fluids $\tau = \tau_0 + \eta \frac{dV}{dr}$, it results that

$$\frac{\partial \tau}{\partial r} + \frac{\tau}{r} = \frac{\partial p}{\partial z} \tag{10}$$

In the sub-field (1), the shear velocity is high and the fluid tends to have a Newtonian behavior.

In the sub-field (2), the fluid moves with a constant velocity as a solid plug without being subject to shear.

The relations of the flow velocities in the two areas under the action of the pressure gradient are determined by applying the limit conditions (barrier) for the flow velocity V.

The solution of the Eq. (1) is:

$$V(r) = \frac{Dp}{4 \cdot \eta \cdot l} r^2 + \frac{\tau_0}{\eta} r + c_1 \ln r + c_2 \tag{11}$$

For $r = r_0$ (plug barrier) $\frac{dV(r)}{dr} = 0$

$$\frac{r_0^2 \cdot Dp}{2 \cdot \eta \cdot l} + \frac{\tau_0 \cdot r_0}{\eta} + c_1 = 0 \tag{12}$$

The c_1 constant results and $V(r)$ has the expression:

$$V(r) = \frac{Dp}{4 \cdot \eta \cdot l} r^2 + \frac{\tau_0}{\eta} r - \frac{Dp}{2 \cdot \eta \cdot l} r_0^2 \ln r - \frac{\tau_0}{\eta} r_0 \ln r + c_2 \tag{13}$$

For $r = R$ (at the wall) $V(R) = 0$:

$$\frac{Dp}{2\cdot\eta\cdot l}\left[\frac{R^2}{2}-r_0^2\ln R\right]+\frac{\tau_0}{\eta}\left[R-r_0\ln R\right]+c_2=0$$

The velocity, $V(r)$ $r\in[r_0;R]$ has the expression

$$V(r)=\frac{Dp}{2\cdot\eta\cdot l}\left[\frac{r^2}{2}-\frac{R^2}{2}+r_0^2\ln\frac{R}{r}\right]-\frac{\tau_0}{\eta}\left[r-R+r_0\ln\frac{R}{r}\right] \tag{14}$$

For determining the r_0 radius of the plug, we consider inside the pipe a cylinder of r_0 radius being in equilibrium under the action of the pressure and friction forces (Fig. 1).

From the equation of the dynamic equilibrium of forces, $(p_1-p_2)\pi r_0^2=2\tau_0 r_0\pi l$ the following expression results for the plug radius:

$$r_0=\frac{2\tau_0 l}{Dp} \tag{15}$$

The traveling velocity V_0 of the fluid plug is obtained from the relation of the velocity $V(r)$ with the condition $r=r_0$

$$V_0=\frac{Dp}{2\cdot\eta\cdot l}\left[\frac{r_0^2}{2}-\frac{R^2}{2}+r_0^2\ln\frac{R}{r_0}\right]-\frac{\tau_0}{\eta}\left[r_0-R+r_0\ln\frac{R}{r_0}\right] \tag{16}$$

4. Theoretical Results Obtained by Numerical Simulation

The numerical modeling was carried out for a *MRF* 132 magnetorheological fluid, Fig. 1 in the following conditions:

Pipe diameter D = 0.025 m;

Pipe length L = 0.015 m;

Pressure difference of the flow: $DP1$ = 12 KPa, $DP2$ = 15 KPa,

Magnetic induction B [0 – 0.4] T

Fluid viscosity dynamic coefficient η = 0.112 Pas

Shear velocity $\gamma=\frac{dV(r)}{dr}$ [s^{-1}] $\gamma=1000$

$\tau_0 = 39.721B^4 - 132.358\,B^3 + 119.0925\,B^2 + 10.280\,B + 0.10815$

the deformation for the MR132 fluid, τ_0 , [KPa] (Han *et al.*, 2009; Hong *et al.*, 2008).

In Fig. 2, deformation varies depending on the size of the magnetic induction *B*.

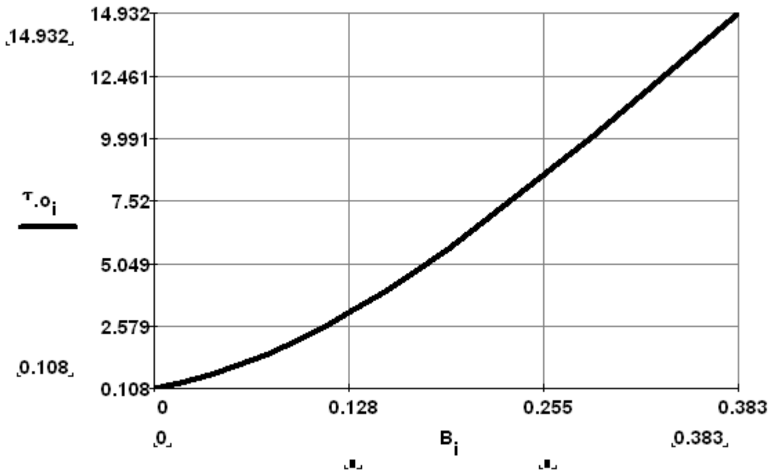

Fig. 2 – Variation of the deformation stress depending on the size of the magnetic induction *B.*

The plastic deformation τ_0 increases as the magnetic induction *B* increases. The increase gradient is higher in high inductions.

Fig. 3 presents the variation of the fluid plug radius for two constant pressure values of the flow, depending on the magnetic induction *B* value.

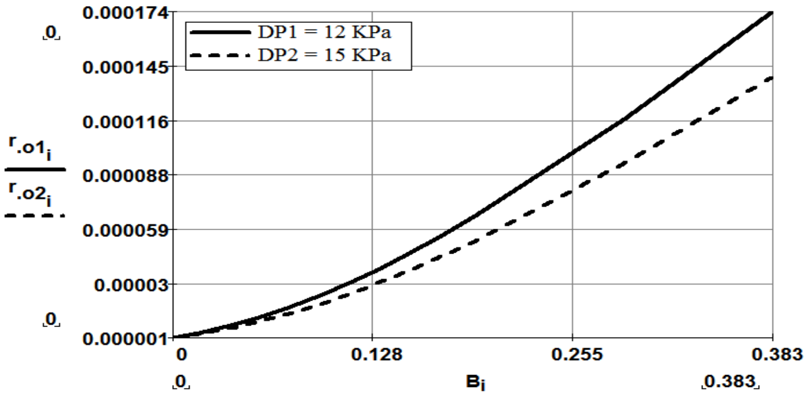

Fig. 3 – Variation of the fluid plug radius.

The r_0 radius of the fluid plug increases as the magnetic induction *B* increases at a constant pressure difference. The increase is determined by the size of the stress τ_0 .

At constant values of the magnetic induction B, the r_0 radius of the fluid plugs decreases as the pressure difference increases.

Fig. 4 presents the velocity variation of the fluid plug for two constant pressure values of the flow, depending on the size of the magnetic induction B.

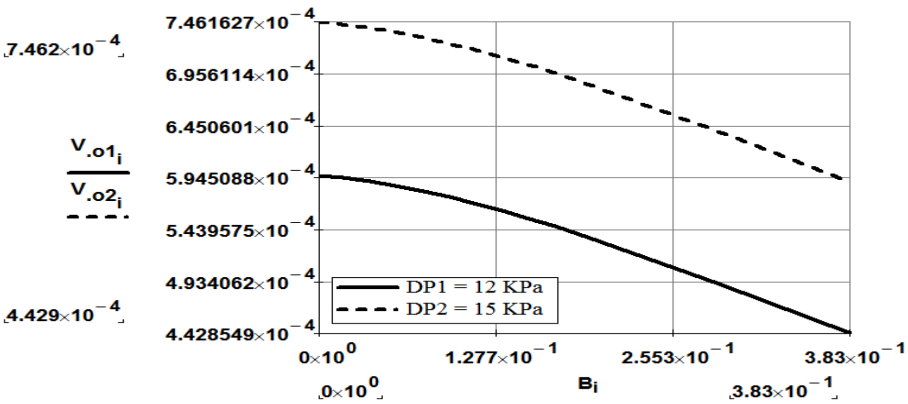

Fig. 4 – Variation of the fluid plug velocity.

The travel velocity V_0 of the fluid plug decreases as the magnetic induction B increases, at constant values of the pressure difference.

At constant values of the magnetic induction B, the V_0 decreases as the pressure difference determining the flow increases.

Figs. 5 and 6 present the variation of local flow velocity $V(r)$ of the magnetorheological fluid in relation to radius r for two constant pressure values of flow, depending on the size of the magnetic induction B.

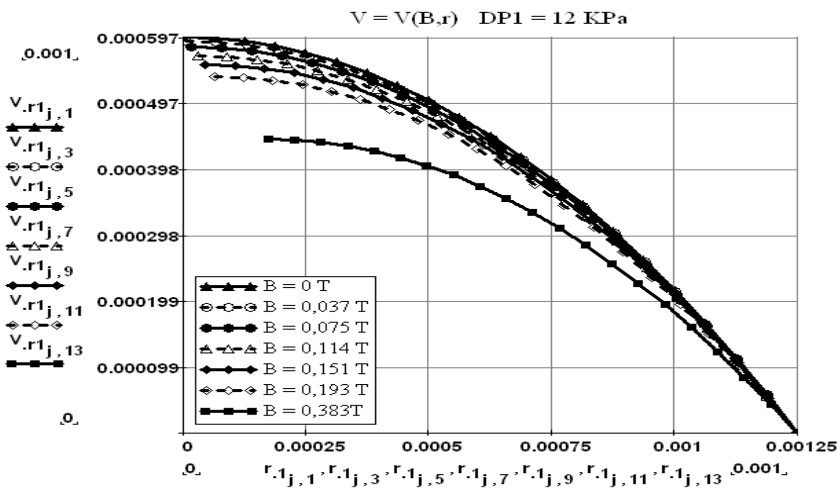

Fig. 5 – Variation of the local velocity $V(B,r)$.

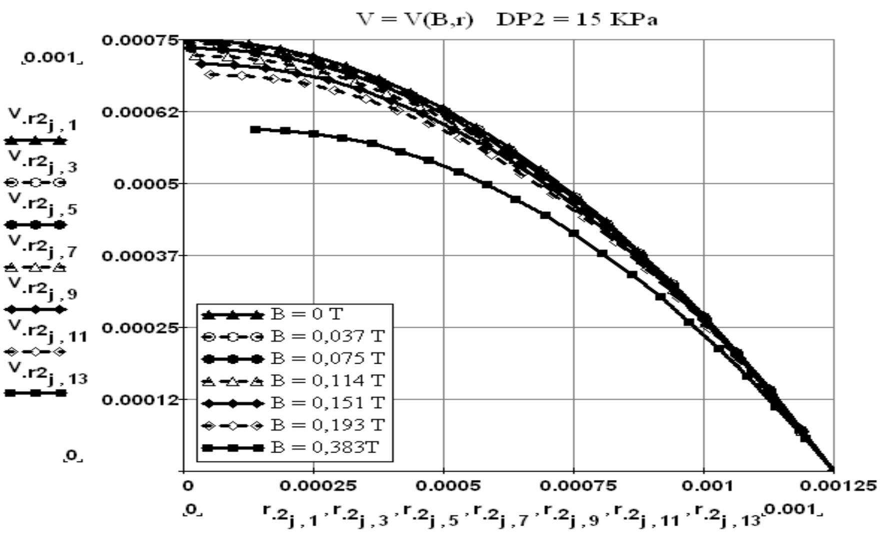

Fig. 6 – Variation of the local velocity $V(B, r)$.

The local flow velocity $V(r)$ in the space between the fluid plug and the pipe wall has an approximately parabolic variation in relation to current radius r and decreases as the magnetic induction B increases.

For a magnetic induction B = *cst.*; r = *cst*. the local velocity $V(r)$ increases as the pressure difference increases.

5. Conclusions

The rheological character of the fluid is strongly influenced by the magnetic induction value.

The r_0 radius of the fluid plug increases as the intensity of the applied magnetic field rises, and for constant magnetic induction values, it decreases as pressure difference rises.

The travel velocity V_0 of the fluid plug is influenced both by the magnetic induction value and by the pressure difference of the flow.

The local flow velocity of the fluid $V(r)$ in the space between the fluid plug and the pipe wall has an approximately parabolic variation in relation to the current radius r. At a constant pressure difference, the velocity $V(r)$ decreases as the magnetic induction B increases.

For a magnetic induction of B = *cst.*, at the radius r = *cst.*, the velocity $V(r)$ increases as the pressure difference increases.

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CURGEREA LAMINARĂ A FLUIDULUI MAGNETOREOLOGIC PRINTR-UN TUB DE CURENT CU SECȚIUNE CIRCULARĂ

(Rezumat)

Fluidele magnetoreologice sunt nenewtoniene de tip Bingham. În cazul mișcării acestora, caracteristicile cinematice și energetice ale curgerii diferă de cele ale fluidului Newtonian. Lichidele magnetoreologice sunt caracterizate de faptul că energizarea acestora se realizează prin intermediul unui câmp magnetic exterior care influențează efortul de alunecare. Specific mișcării fluidelor magnetoreologice de tip Bingham este formarea unui dop fluid în zona centrală care se deplasează cu viteză constantă. Lucrarea are ca scop analiza influenței câmpului magnetic exterior aplicat fluidului magnetoreologic asupra vitezei de curgere a acestuia. Modelarea numerică a fenomenului, arată că mărimea câmpului magnetic are influență considerabilă asupra vitezei de curgere a fluidului.